Discovery learning – constructivistic approach using Mathematica as a cognitive tool: Which form of application is more effective in university Mathematics' courses?

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Abstract
In this paper the Discovery learning-constructivistic approach is analysed, based on the ideas of Discovery learning and Constructivism, in university Mathematics' courses when Mathematica is used as a cognitive tool. A case study is presented that has been realized in the Mathematics' Department of the University of Athens, concerning the most effective form of application of the approach amongst two programs that were applied: The first program, realized partly in the classroom and partly in the computer laboratory, included a resuming-reviewing program in the laboratory that followed the traditional theoretical teaching program in the classroom and included the negotiation of properties, geometric interpretations and applications of the theoretical subjects by the students using Mathematica themselves. The second program, realized wholly in the computer laboratory, was orientated in the practical negotiation of the subjects mentioned above using Mathematica, taking turns with the theoretical negotiation which was limited to the minimum. The paper aims to compare the two forms of application on their impact on students' scores and on the qualitative characteristics of the teaching-learning process and conclude on the most effective form. The results of the statistical analysis indicate that even though both programs were effective, the first program was more effective concerning students' scores, the students' active participation in the lesson and the extent of presentation by the teacher.

Keywords: Higher Education, Mathematics' Education, Discovery Learning, Constructivism, Cognitive tools

JEL classifications: I20, I23

Introduction

Contemporary research in Education points out the significance of the students' active participation during the lesson, their self-action, exploration and experimentation and the cooperation of the students with one another and the teacher, characteristics that come in contrast to the traditional teaching approach in higher education, the narrative approach (lectures). Also the significance of social learning is highlighted via the cooperation of the students with one another and with the teacher. Discovery learning approaches, based on the ideas of Discovery learning and Constructivism, are proposed as able to create the appropriate active, creative and explorative environment in Mathematics' university courses (Korres, 2007).

Moreover Information Age has changed the needs of today's citizens, who are systematically trained and further educated. Businesses and organizations are developed and reorganized. Employees who take initiative and introduce diverse points of view are in great demand. Communications are conducted via networks and emphasis is laid on customization. Also technology is rapidly and constantly developed and initial education and training proves shortly to be inadequate (Kyriazis, Psycharis & Korres, 2009). Discovery learning and constructivistic learning educates students to learn how to learn, a quality that can prepare and qualify today's students (who are tomorrow's employees, businessmen, administrators, educators etc) to adjust to the future societies' needs and attributes.

In this paper Discovery learning-constructivistic approach using Mathematica as a cognitive tool in Mathematics' university courses is analysed. The role of computers in the contemporary educational environment has changed. Computers are used as cognitive tools, computer-based tools that have been developed or adjusted to function as "intellectual partners" of the students (Korres, 2007).

Also a case study is presented concerning the most effective form of application of the teaching approach. The case study has been realized in the Mathematics' Department of the University of Athens. The Discovery learning-constructivistic approach using Mathematica as a cognitive tool included the negotiation of properties, geometric interpretations and applications of the theoretical subjects in two forms of application: a) A resuming-reviewing program in the computer laboratory, that followed the traditional theoretical program of teaching realized in the classroom and b) A program orientated in the practical negotiation of the subjects, taking turns with the theoretical negotiation, realized wholly in the computer laboratory. The paper' main goal is to compare the two forms and conclude on the most effective on their impact on students' scores and on the qualitative characteristics of the teaching-learning process.

The mathematical object selected is Differential Geometry of curves and surfaces, in particular the theory of plane and space curves (Abbena, Salamon & Gray 2017, Henderson, 1998), mainly because it has many interesting and useful applications in many mathematical fields and fields that are not directly related to Mathematics, it addresses to both students of departments of Mathematics and departments of Science, Polytechnic schools and Economic Schools and it unifies many mathematical fields' objects as Analytic Geometry, Calculus and Linear Algebra. Moreover, although Geometry has always aided intuition in econometrics, more recently Differential Geometry has become a standard tool for econometrics, as it is used in the analysis of statistical models (Marriott & Salmon, 2000).

The software selected as a cognitive tool is Mathematica (Torrence & Torrence, 2009, Abbena, Salamon & Gray 2017), mainly because its mathematical operations' notation and objects are similar to the standard mathematical notation, it has a function-based structure which allows us to define and study geometric objects and quantities as real functions of real variables and it offers possibilities in plotting graphs easily, quickly and precisely and in making complex calculations quickly and accurately.

Mathematica is commercial software. However the majority of university departments of Mathematics and departments of Science, Polytechnic schools and Economic Schools have a number of licences of the software for use on campus, also there is a free online version of the software at the address: http://develop.open.wolframcloud.com/app/

The free online version of Mathematica requires registration in order for the user to save and open notebooks. The free online version can be used at the lessons if the department does not have the software and by the students at home or on campus.

The structure of the paper is as follows: In section 2 we discuss the ideas upon which Discovery learning-constructivistic approach using Mathematica as a cognitive tool is based, in section 3 we describe the approach itself, in section 4 we present the methods used in the case study, in section 5 the results of the case study are presented and in section 6 we discuss the results giving some concluding remarks.

Literature review

Discovery learning

Bruner (1960, 1966) proposed Discovery learning in the teaching and learning of Mathematics. The teacher's basic role is to help and encourage his/her students to discover the mathematical concepts and ideas. Discovery is a process and a general attitude of exploration and experimentation. The idea of discovery is also supported by Jean Piaget who noted "apprendre c' est inventer" (to learn means to discover) (Korres, 2007).

According to Bruner's theory, the teacher is the facilitator of the students' learning. As the facilitator, he has the opportunity to go around the classroom and answer students' questions as they apply the information they learned to the lectures and as they work with their fellow students. This process promotes active learning, which contributes to students' creativity and problem-solving skills. The students working in groups, can review case scenarios and apply the information and methods they are taught in the form of solutions to problems identified (Costello, 2017).

Constructivism

Piaget's ideas have set the foundations for Constructivism (Sinclair, 1987, Steffe & al., 1988). According to Constructivism, the student constructs knowledge actively, using his/her preexistent knowledge. Knowledge cannot be transmitted or transferred to students by the passive acceptance of the views and ideas proposed by the teacher. The term "Constructivism" was introduced by Papert, who gave emphasis on the participation of the learner to the process of learning and to the way the learner himself constructs the mental "map" via which he percepts, processes and understands that process (Papert, 1980, 1993).

Learning is activated by action on problematic situations (Thompson, 1985). The student, when he/she faces a problem or a situation that he/she cannot explain, interpret or solve with his/her preexistent cognitive structures, is lead to unsteadiness or lack of balance. As a result of this imbalance, student's previous conceptions or ideas are modified, in order to deal with and incorporate the new experience.

A wide collection of models of teaching and instructional design theories and models based on the theories of Discovery learning and

Constructivism are described and analysed in Joyce, Weil & Calhoun (2015) and Reigeluth (1999).

Computers as cognitive tools

Scaffolding provides support to learners in narrowing the gap between what they can do themselves and what they can do under guidance. Software-realized scaffolding includes conceptual aspects of scaffolding and different scaffolding techniques implemented in software. A very powerful idea is the concept of computers as cognitive tools that can be used as "intellectual partners" of the students in creating scaffolding towards meaningful thinking (Korres, Psycharis & Makri-Botsari, 2011).

Cognitive tools or mindtools are learning environments and computer-based tools that have been developed or adjusted, in order to function as intellectual partners of the students, in order to activate and accommodate critical thinking and higher order learning (Jonassen, 2000). Researchers of Instructional technology, both those who study intelligent tutoring/artificial intelligence technologies and those who propose a constructivist/developmental perspective that promotes exploration and social interaction, view computer learning environments as cognitive tools that can enhance learning, performance and understanding (Lajoie & Derry, 2013).

According to Jonassen (2000) cognitive tools are generalizable computational tools that support knowledge construction and transferable learning. They have simple, powerful formalism and they are easily learnable. Cognitive tools reorganize (radically reconstruct) the way learners think. They support, guide and extend the thinking processes of their users. They aim in activating and facilitating the cognitive process.

Discovery learning-constructivistic approach using Mathematica as a cognitive tool

Discovery learning-constructivistic approaches are based on the ideas of Jerome Bruner for Discovery Learning and the ideas of Constructivism, as stated and explained above. Discovery learning approaches enable students to accomplish results for which they did not possess a formed algorithm, via a process of exploration and experimentation, without having those results posed or explained to them.

The teaching-learning process of discovery learning-constructivistic approaches includes the following steps (adapted from Korres, 2007):

1 Definition of the subject under negotiation or the problematic situation.
2 Gathering of data, elaboration, organization and analysis of data.
3 Formation and formulation of conjectures via experimentation.
4 Checking of the conjectures.
5 Formation and formulation of conclusions via experimentation.
6 Formation and formulation of general conclusions by the community of the class via discussion.
7 Investigation, discussion and generalization of the conclusions.
8 Summarization of the main points of the lesson.
9 Reflection on the subjects taught and on the process of teaching and learning.

Discovery learning approaches have many significant advantages: They create an energetic environment in classroom, as the students participate actively and creatively in the teaching–learning process. The students have the opportunity to participate in a collective project in which they gather and organize data, get acquainted with their co–students' ideas and practice communication skills. Learning accomplished via discovery learning approaches is essential and not mechanical and can be effectively used by the student in other domains, since the student participated actively in its formation. Discovery learning approaches cultivate students' self-confidence that can act as a powerful motive for further learning, in order for the student to satisfy his personal investigating interest and not for utilitarian reasons (for example in order to increase his grades) (Korres, 2007, 2008).

Computer based cognitive tools as Mathematica can be used effectively as means of instruction by the educator or as interactive means of self–instruction by the students. Students can use Mathematica in order to explore and experiment with the learning material. They can use Mathematica in the formation, checking and investigation of their conjectures and hypotheses. Mathematica can facilitate the teacher and the students by freeing them from tiresome, uncreative tasks such as difficult, complex calculations when the lesson's tasks do not include learning how to perform calculations. It can accommodate the teacher and the students in plotting complex graphs easily, quickly and precisely. It can be used for the visualization of geometric quantities, concepts and models that are difficult or impossible to be understood otherwise (Korres, 2007, 2008).

Methods

Description of the study

The case study presented in this paper is part of a broader research project carried out by the author, studying Instructional design issues in Mathematics' and Sciences' Higher Education using computers. Studies have been realized at the Mathematics' Department of the University of Athens, at the Department of Statistics and Insurance Sciences of the University of Piraeus and at the Department of Education of ASPETE.

The case study presented in this paper was realized in the Mathematics' Department of the University of Athens. A total of 59 students participated, in two groups comprising of 25 and 34 students respectively (Group A and Group B). Group A and Group B comprised of the students enlisted in the courses "Subjects of Algebra and Geometry II" and "Geometry for Didactics of Mathematics" that attended the lessons regularly. These two courses have common parts in their Syllabus, so the teaching of both groups' students was limited to those common subjects, in order for both groups to be taught the same subjects using a differentiated form of the Discovery learning approach in each group.

The two groups were checked on their equivalence in the students' competence in Mathematics before the interventions. Since students' enrolment in the two courses was based on their scores in previously taught courses (as Calculus I, Calculus II and Linear Algebra), competence of the students in Mathematics can be regarded as not affecting the result of the study.

Also the two groups were checked on their equivalence in the students' practical engagement in computers' use and the students' interest in using computers. Also the teacher who taught the two courses was the same. In this way factors as interest and practical engagement in computers' use and individual competences of the teacher can be regarded as not affecting the result of the study.

The differentiated program of teaching

Group A's program of teaching included two parts: The first part included traditional teaching of the theoretical subjects via the narrative approach in conventional classroom settings at a percentage of 60% of the total number of lessons. The second part included the teaching of properties, geometric interpretations and applications of the theoretical subjects via discovery learning approach using Mathematica in the computer laboratory, with the students operating on the computers at a percentage of 40% of the total number of lessons. Group A's students were also assigned with personal assignments which they had to prepare using Mathematica.

Group B's program of teaching was orientated in the practical negotiation of concepts with the use of Mathematica and was realized wholly in the laboratory with the students operating on the computers. The study of the theoretical subjects was realized via the narrative approach and was limited to the minimum; the study of properties, geometric interpretations and applications of the theoretical subjects was realized mainly via discovery learning-constructivistic approach using Mathematica; the two forms of teaching were used taking turns. The program was realized wholly at the computer laboratory.

Both Group A and Group B's programs of teaching made use of the traditional narrative approach and the discovery learning-constructivistic approach using Mathematica. The main difference between the two groups' programs is that Group A attended the traditional theoretical negotiation of the concepts being taught in the classroom, whereas Group B attended only the reference of the theoretical subjects without proofs and with limited theoretical negotiation in the computer laboratory. Group B's students attended more courses in the laboratory and a program of teaching which made use of more sophisticated programs and commands of Mathematica than the program of Group A' s students. Group A' students had to prepare personal assignments using Mathematica.

The students of both groups were introduced to using Mathematica, getting acquainted with the notation of the basic mathematical symbols, operations and objects in Mathematica and the definition of functions and quantities as functions, how to plot the trace of a curve in plane and in space using Mathematica and how to represent vectors in plane. During these lessons the students were let to experiment by plotting traces of various curves or vectors that were proposed by the teacher or were selected by them, in order to become familiar with the structure and the options of the commands of the program.

The discovery learning-constructivistic approach using Mathematica for both groups was realized in the computer laboratory in a series of two hours' lessons. The teacher could write on the board the elements of theory that he thought as necessary, the commands and the programs of Mathematica that the students needed and the conjectures, hypotheses and conclusions that were formulated by the students. Both groups' students were divided in groups of 2 students per computer. They were cooperating with the members of their groups, with the members of other groups and with the teacher in discovering the answers to the problematic situations that were posed in the classroom. They had the opportunity to ask the help of the teacher at any time, relating to the understanding of the elements of theory that were mentioned and the use, syntax and function of the commands of Mathematica. They also had the opportunity to set into discussion questions, conjectures and conclusions to the community of the class.

The methodology of the study

The main research question that this paper aims to answer is "which form of application of the Discovery learning-constructivistic approach using Mathematica is more effective concerning its impact on students' scores and on the qualitative characteristics of the teaching-learning process"? Or equivalently, having chosen to apply the Discovery learning-constructivistic approach using Mathematica in a university Mathematics' course, "do we choose to teach it wholly in the computer laboratory or do we teach the theoretical subjects of the course in conventional classroom settings for 60% of the lessons, we use the computer laboratory for 40% of the lessons and we hand out personal assignments to the students that they have to prepare using Mathematica"?

The research approaches used are the descriptive-investigative approach, the experimental approach and the correlational approach (Cohen, Manion & Morisson, 2013). The students that participated in the study were a convenient sample, that is why the study is characterized as a case study and the results can be generalized only for groups of students with similar characteristics.

A questionnaire was designed and developed in order to evaluate the two forms of application of the approach. The development of the questionnaire concerns the evaluation of the students' practical engagement on computers' use and on their interest in using computers, the evaluation of the students' understanding of the concepts having being taught (knowledge-skills test) and the evaluation of the students' views on the qualitative characteristics of the teaching-learning process.

The data that was gathered by the encoding of the questionnaires, was analysed via the statistical program SPSS. The use of non parametric criteria was regarded as more appropriate for the comparison of the two groups concerning students' scores and students' views on the qualitative characteristics of the teaching-learning process, since the conditions of parametric criteria are not satisfied (sizes of the two groups 25 and 34 students respectively and the tests for normality showed that the two groups' distributions were not normal for all variables). The level of significance chosen in all tests is 5%.

Results

The students that participated in the study

The students that participated in the study were 59. As for the gender, 31 students were males (52.5%) and 28 were females (47.5%). Group A comprised of 25 students, 13 of them were males (52%) and 12 females (48%). Group B comprised of 34 students, 18 of them were males (52.9%) and 16 females (47.1%). As we can see an approximate gender balance was maintained in both groups.

The two groups' students were checked on their practical engagement in computers' use and on their interest in using computers. The results are shown in Table 1.

Table 1: Results on the students' practical engagement in computers' use and on their interest in using computers

	Group A		Group B		Total	
	Yes	No	Yes	No	Yes	No
1. Are you using computers at the University?	76.0%	24.0%	82.4%	17.6%	79.7%	20.3%
2. Are you using computers at home?	76.0%	24.0%	70.6%	29.4%	72.9%	27.1%
3. Do you find interest in using computers?	88.0%	12.0%	91.2%	8.8%	89.8%	10.2%

The majority of the two groups' students made use of computers either at the University (76% and 82.4% respectively) or at home (76% and 70.6% respectively). As we can see an approximate balance was maintained in both groups concerning the students' practical engagement in computers' use.

Also the majority of the two groups' students (88% and 91.2% respectively) stated that they find interest in using computers. In order to check whether the two groups can be considered equivalent concerning the students' interest in using computers we used X^2–Homogeneity test. The results of the test did not show a significant statistical difference between the two groups (X^2–Homogeneity, $X^2=0.159$, $p=0.690$, $Df=1$).

Evaluation of the students' understanding of the concepts being taught

The evaluation of the students' understanding of the concepts being taught was based on the results of a knowledge-skills test that was given to the students after the completion of the two programs. The test was answered by 53 students; the whole of Group A's students and 28 of Group B's students.

In order to measure the level of understanding of the students that participated in the research, we made an analysis of the students' scores in the test. The students ranked from 3 to 9 in a 1-10 scale. The mean grade was 6.68 with std. deviation 1.81. The level of the students' understanding can be considered as very high, as 60.4% ranked above 7 and only 11.3% failed the test.

Moreover we made a statistical analysis for the two groups individually concerning the students' level of understanding. The analysis showed that Group A had mean grade 6.80 with std. deviation 1.80 and Group B had mean grade 6.57 with std. deviation 1.83. Group A's percentages are higher for higher students' scores, as 76% ranked above 7, compared to 46.4% of Group B's students. The two groups' students present similar percentages in failing the test (12% and 10.7% respectively) (see Table 2 and Figure 1). The former results are supported by X^2-Homogeneity test according to which the two groups are not homogenous concerning the students' scores (X^2-Homogeneity, X^2=13.396, p=0.020, Df=5).

Table 2: Distribution of students' scores

Scores (/10)	Group A	Group B	Total
3	12%	10.7%	11.3%
4	0%	0%	0%
5	12%	10.7%	11.3%
6	0%	32.1%	17%
7	36%	14.3%	24.5%
8	28.0%	10.7%	18.9%
9	12.0%	21.4%	17.0%
Total	100%	100%	100%

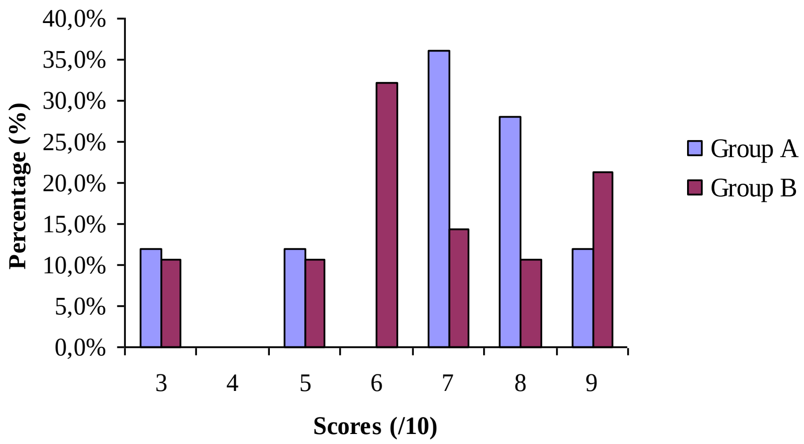

Figure 1: "Distribution of students' scores

Evaluation of the students' views on the qualitative characteristics of the teaching-learning process

The results of the evaluation of the students' views on the qualitative characteristics of the teaching-learning process regarding the two forms of the Discovery learning-constructivistic approach using Mathematica are presented in Table 3.

Table 3: Results on the students' views on the qualitative characteristics of the teaching-learning process

	Group A			Group B			Total		
	Yes, in a great extent	Yes, in some extent	No	Yes, in a great extent	Yes, in some extent	No	Yes, in a great extent	Yes, in some extent	No
1. Did the approach provoke your interest for the lesson?	88%	12%	0%	90.3%	9.7%	0%	89.3 %	10.7%	0%
2. Did the approach encourage your active participa-tion in the lesson?	100%	0%	0%	73.5%	26.5%	0%	84.7 %	15.3%	0%
3. Did the approach encourage you to self-act, explore and experiment?	64%	36%	0%	73.5%	17.6%	8.8 %	69.5 %	25.4%	5.1%
4. Did the approach encourage the formulation and checking of conjectures?	86.4 %	0%	13.6 %	80.6%	19.4%	0%	83%	11.3%	5.7%
5. Did the approach encourage the discussion and conversation ?	72.7 %	27.3%	0%	82.4%	8.8%	8.8 %	78.6 %	16.1%	5.4%

The approach provoked the students' interest for the lesson, as stated by the whole of the students that answered the corresponding question; indeed 89.3% in a great extent. Group A and Group B are homogenous in point of the extent of interest for the lesson (X^2–Homogeneity, X^2=0.078, p=0.780, Df=1). The two groups accomplished only positive answers and approximately the same percentages in the greater extent (Group A: 88%, Group B: 90.3%).

The whole of students stated that the approach encouraged their active participation in the lesson and 84.7% in a great extent. The ways in which this was accomplished, according to the students' observations, are:

- Experimentation with Mathematica and the possibilities it offers.
- Possibility of experimentation and investigation with the concepts.
- Formulation of conjectures and conclusions and discussion on the process of solving the problematic situations that were studied in the classroom.

- Direct confrontation with the questions and difficulties that came along during the course of the lesson (feedback).

Group A and Group B disagree on the extent of their active participation (X^2–Homogeneity, X^2=7.809, p=0.005, Df=1). The two groups accomplish only positive answers, but Group A shows a higher percentage in the greater extent than Group B (Group A: 100%, Group B: 73.5%).

The approach encouraged the students' self–action, exploration and experimentation, as 94.9% of the students stated; indeed 69.5% in a great extent. The ways in which this was accomplished, according to the students' observations, are more or less the same as the ones stated in the previous paragraph. An interesting observation of Group A's students is that they self–acted, explored and experimented via the preparation of their assignments.

Group A and Group B are homogenous concerning their self-action, exploration and experimentation (X^2–Homogeneity, X^2=4.303, p=0.116, Df=2). Group A shows only positive answers, while Group B has some negative answers (8.8%); however Group B presents a higher percentage in the greater extent than Group A (Group A: 64%, Group B: 73.5%). A noteworthy observation is that Group A' students self–acted in the laboratory in a limited number of lessons, but also via the preparation of their assignments, while Group B self–acted in the computer laboratory in a large number of lessons. We can formulate the conjecture that in a time–limited application of the discovery learning approach, we can equalize the gains of self–action handing out personal assignments to the students, which they can prepare working with the cognitive tool.

The teaching approach also encouraged the formulation and checking of conjectures concerning the discovery of the properties and rules that were discussed in the classroom, as stated by 94.3% of the students that answered the corresponding question; indeed 83% in a great extent. The ways via which this was accomplished, as stated by the students, are:

- Experimentation with the program.
- Practical negotiation of the concepts being taught.

The two groups are not homogenous concerning the students' views on formulation and checking of conjectures (X^2–Homogeneity, X^2=8.536, p=0.014, Df=2). Group B shows only positive answers, while Group A shows negative answers (13.6%); the percentage of Group A in the greater extent is slightly higher than the percentage of Group B (Group A: 86.4%, Group B: 80.6%)

The teaching approach encouraged the discussion and conversation between the students and the teacher and the students with one another, as stated by 94.6% of the students that answered the corresponding question; indeed 78.6% in a great extent. The students point out the following ways:

- Comparison of results and conjectures that result from personal experimentation.
- Verification of what is taught in the classroom.

The two groups are homogenous concerning the students' views on whether the teaching approach encouraged the discussion and conversation in the classroom (X^2-Homogeneity, $X^2=4.928$, $p=0.085$, $Df=2$). Group B shows a higher percentage in the greater extent than Group A (Group A: 72.7%, Group B: 82.4%), but also some negative answers (8.8%), while Group A shows only positive answers.

The extent of presentation by the teacher was characterized mainly as very thorough by Group B's students (82.4%) and as sufficiently thorough (64%) by Group A's students. The two groups do not have the same distributions of answers concerning the characterization of the presentation by the teacher (Mann-Whitney, $U=141$, $p<0.001$). We should note however that the extent of presentation by the teacher in discovery learning-constructivistic approaches should be sufficiently thorough but not very thorough. Therefore Group A's program can be considered as a more effective application concerning that matter.

Regarding the guidance of the students by the teacher for the discovery of the properties and rules that were discussed in the lesson, the majority of the students, in particular 88% of Group A's students and 79.4% of Group B's students, characterized the guidance as sufficient towards the tasks of the lesson; the rest of the students characterized the course of instruction as completely guided by the teacher. The two groups have the same distributions of answers in point of their characterization of guidance of the students by the teacher (Mann-Whitney, $U=388.5$, $p=0.389$). We should note that the level of guidance by the teacher, in discovery learning-constructivistic approaches, should be guidance to the tasks of the lesson, therefore both forms of application were as effective concerning that matter.

The advantages of Discovery learning approaches using Mathematica as a cognitive tool compared to the traditional narrative approach, as stated by the students of the two groups, are:

- Active participation of the students in the lesson.
- Possibility of personal discovery of the subjects being taught.
- Possibility of self-action and experimentation with the use of the program.
- Personal construction of knowledge by the student.

The disadvantages of Discovery learning approaches using Mathematica as a cognitive tool compared to the traditional narrative approach, as stated by the students of the two groups, are:

- Requirement of more time compared to the traditional instruction.
- There is the risk for the student to neglect the value of the process of proving.
- There is the risk for the process of teaching-learning to become teacher-centered.

Discussion / Conclusions

The students that participated in the study had the opportunity to experience the narrative approach and the Discovery learning-constructivistic approach using Mathematica as a cognitive tool. Group A attended the traditional program of teaching, via the narrative approach in the classroom and a second program of teaching which included the negotiation of practical subjects concerning the theoretical subjects being taught, via Discovery learning-constructivistic approach using Mathematica in the computer laboratory. Group B's program was orientated in the practical negotiation of the concepts and was realized wholly in the laboratory; the two forms of teaching were used taking turns with the theoretical negotiation of the concepts limited to the minimum.

In Table 4 summative results are presented on the two groups' differences concerning the students' scores and the students' views on the qualitative characteristics of the teaching-learning process.

Table 4: Results on the two groups' differences on students' scores and the students' views on the qualitative characteristics of the teaching-learning process

	X^2-Homogeneity test	**Mann-Whitney test**	**Which group's form of application was more effective?**	**Results**	
				Group A	**Group B**
Interest in using computers	Homogenous (p=0.690)	-	**Equivalence**	Vast majority positive answers	
Students' scores	Not homogenous (p=0.020)	-	**Group A**	76% above 7; 12% failed the test	46.4% above 7; 10.7% failed the test
Interest for the lesson	Homogenous (p=0.780)	-	**Equivalence**	Only positive answers; Vast majority greater extent	
Students' active participation	Not homogenous (p=0.005)	-	**Group A**	Only positive answers in the greater extent	Only positive answers; Greater extent: 73.5%
Students' self-action, exploration and experimentation	Homogenous (p=0.116)	-	Not a safe conclusion	Only positive answers	Some negative answers; Higher percentage in the greater extent
Formulation and checking of conjectures	Not homogenous (p=0.014)	-	Not a safe conclusion	Some negative answers; Slightly higher percentage in the greater extent	Only positive answers
Discussion and conversation	Homogenous (p=0.085)	-	Not a safe conclusion	Only positive answers	Some negative answers; Higher percentage in the greater extent
Presentation by the teacher	-	Not the same distributions of answers (p<0.001)	**Group A**	Sufficiently thorough: 64%	Very thorough: 82.4%

Guidance by the teacher	–	Same distributions of answers (p=0.389)	**Equivalence**	Vast majority sufficient towards the tasks of the lesson

The Discovery learning approach can be characterized as effective in both forms of application as indicated by the results of the statistical analysis. The two groups can be regarded as equivalent concerning the students' competence in Mathematics, the students' practical engagement in computers' use and their interest in using computers before the interventions. Also the teacher who taught both groups was the same. Therefore we can conclude that factors as the students' competence in Mathematics, the students' engagement and interest in computers' use and the teacher' individual competences did not influence neither students' scores nor their views on the qualitative characteristics of the teaching-learning process.

The results of the statistical analysis indicate that Group A's form of application was more effective than Group B's in students' scores, in students' active participation in the lesson and in the extent of presentation by the teacher.

The two groups' programs can be considered as equivalent concerning the students' interest for the lesson and their views on the guidance by the teacher while we cannot come to a safe conclusion concerning the students' self-action, exploration and experimentation, the formulation and checking of conjectures and the discussion and conversation.

Based on the conclusions mentioned above, we propose the Discovery learning-constructivistic approach using Mathematica as a cognitive tool in Mathematics' university courses at a program which combines the traditional theoretical negotiation of subjects in the classroom via the narrative approach at a percentage of 60% of the lessons, the practical negotiation of properties, geometric interpretations and applications via the Discovery learning-constructivistic approach using one or more cognitive tools in the computer laboratory at a percentage of 40% of the lessons and the preparation of personal assignments by the students using the cognitive tool or tools, working at hours independent of the teaching hours of the course. In this way we can maximize the gains of the Discovery learning-constructivistic approach and the possibilities cognitive tools can offer, without neglecting the theoretical negotiation of subjects and without needing to ensure the computer laboratory's availability for the 100% of the lessons.

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